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Why should Kemeny's constant be a constant?

Abstract

Consider an irreducible, discrete-time Markov chain $\{X_n : n \geq 0\}$ on the finite state space $\mathcal{S}$. Define T_j as the first return time to state j, for $j \in \mathcal{S}$, and denote by $\boldsymbol{\pi}$ its stationary probability vector. The sum $\sum_{j \in \mathcal{S}} \pi_j \mathrm{E}[T_j | X_0 = i]$ is a constant K, independent of the initial state i. This is proved in Kemeny and Snell's 1960 book on finite Markov chains, and it is named Kemeny's constant.

Why should this sum be independant of i? A prize was offered to the first person to give an intuitively plausible reason, and it was won in 1983 by Peter Doyle.

The quest for simple physical explanations never stops, however, and we offer our own heuristic justification, backed by a renewal argument. Our justification easily extends to Markov chains on an infinite, denumerable state space, and to Markov chains in continuous time.

(Work with Jeffrey Hunter and Peter Taylor)